Linear state estimation of balanced and unbalanced systems

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CÁTEDRA ENDESA DE LA UNIVERSIDAD DE SEVILLA
Escuela Técnica Superior de Ingeniería de Sevilla

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Greetings from Northeastern University

- Location: Boston, Massachusetts, USA
- Web: www.northeastern.edu
- Student population
  - Undergraduate: ~16,000
  - Graduate: ~5000
  - International: 15% (125 countries)
- Colleges and schools
  - College of Arts, Media, and Design
  - College of Business Administration
  - **College of Computer and Information Science**
  - **College of Engineering**
  - Bouvé College of Health Sciences
  - College of Professional Studies
  - College of Science
  - College of Social Sciences and Humanities
  - School of Law

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Outline

- Synchronized Phasor Measurements
- Phasor-Only Linear State Estimation
  - WLS formulation
  - LAV formulation
  - Hybrid (WLS/LAV and SCADA/PMU) estimation
  - Application to 3-phase unbalanced grids
- Closing remarks

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Phasor measurement units (PMU)

"Synchronized Phasor Measurements and Their Applications" by A.G. Phadke and J.S. Thorp

Invented by Professors Phadke and Thorp in 1988.

They calculate the real-time phasor measurements synchronized to an absolute time reference provided by the Global Positioning System.

PMUs facilitate direct measurement of phase angle differences between remote bus voltages in a power grid.

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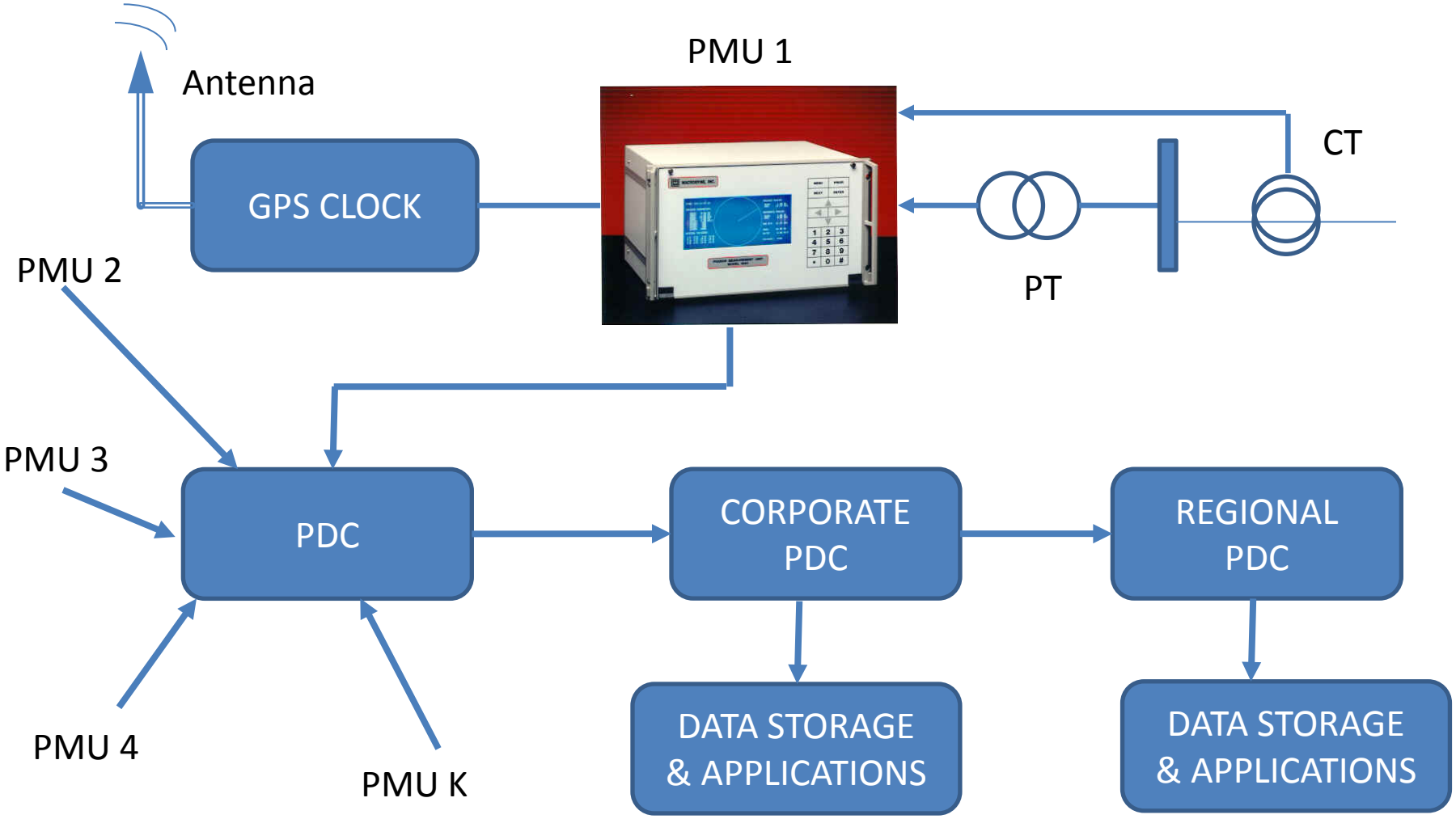
Phasor Measurement Units (PMU)
Phasor Data Concentrators (PDC)[*]



[*] IEEE PSRC Working Group C37 Report

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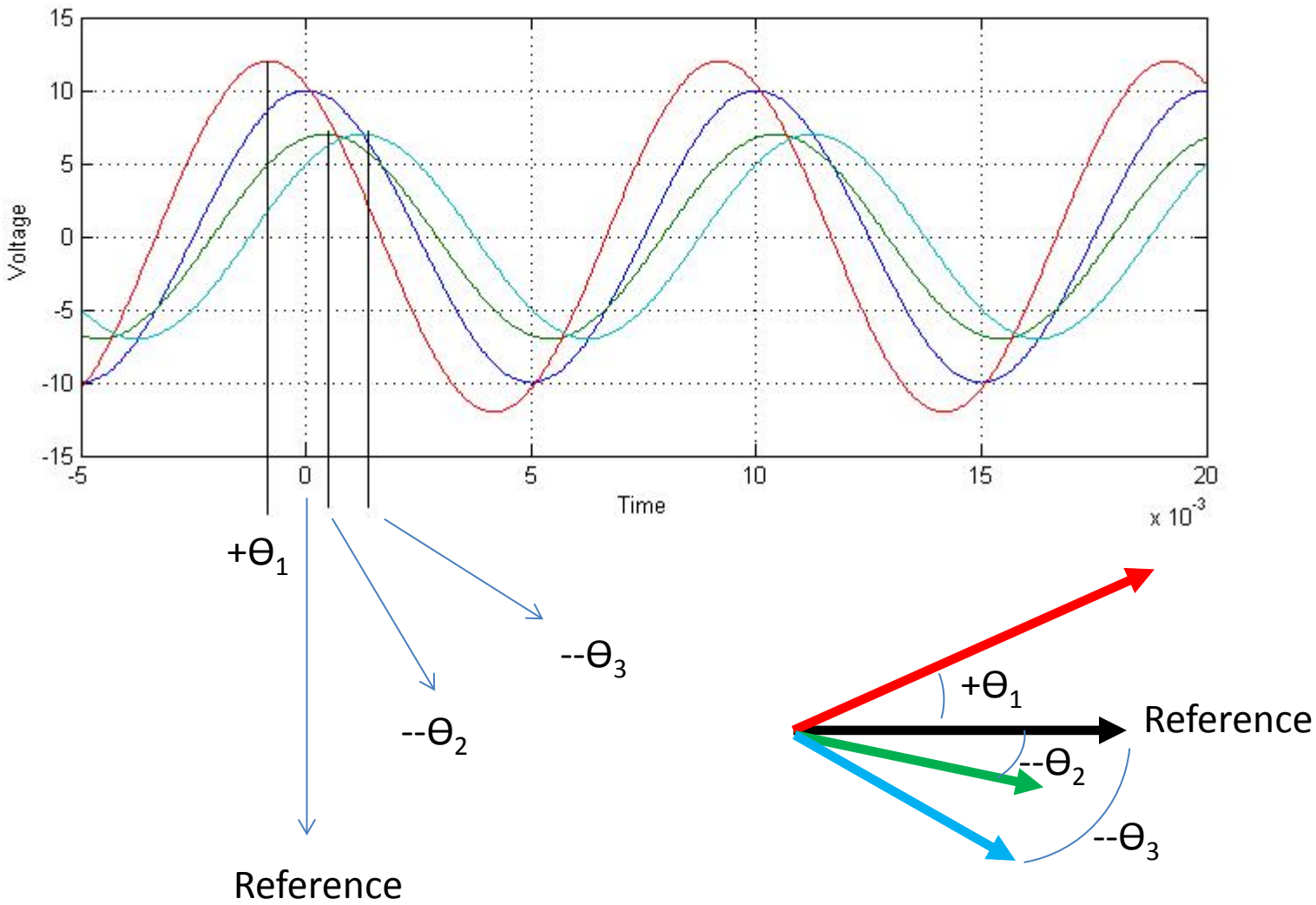
Reference phasor



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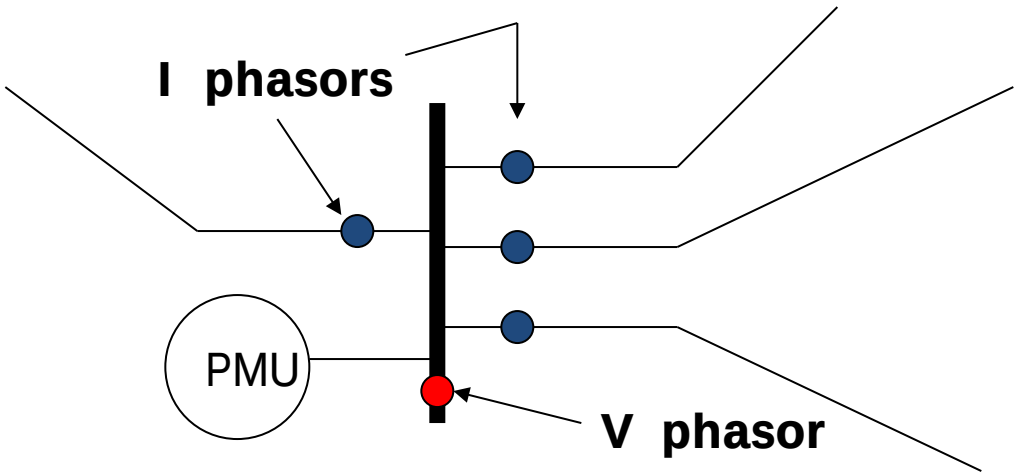
Measurements provided by PMUs



ALL 3-PHASES ARE TYPICALLY MEASURED
BUT
ONLY POSITIVE SEQUENCE COMPONENTS ARE REPORTED

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = [T] \begin{bmatrix} V_0 \\ V_+ \\ V_- \end{bmatrix} \Rightarrow V_+ = \frac{1}{3}[V_A + \propto V_B + \propto^2 V_C]$$

$$\propto = e^{\frac{j2\pi}{3}}$$

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Measurement equations

$$SCADA\ Measurements$$

$$Z = h(X) + \upsilon \quad Non-linear\ Model$$

$$H_x : \nabla h(X)$$

$$Phasor\ Measurements$$

$$Z = H \cdot X + \upsilon \quad Linear\ Model$$

$$H \ : Function\ of\ network\ parameters\ only$$

A.G. Phadke, J.S. Thorp, and K.J. Karimi, “State Estimation with Phasor Measurements”, IEEE Transactions on Power Systems, vol. 1, no.1, pp. 233-241, February 1986.

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Phasor-only WLS state estimation

$$Z = H \cdot X + \upsilon \quad \textit{Linear Model}$$

$$\textit{WLS state estimation problem}:$$

$$\textit{Minimize} \quad \sum_{i}^{m} \frac{r_i^2}{\sigma_i^2}$$

$$\textit{Subject to} \quad r = Z - H \cdot \hat{X} \quad \textit{residual}$$

$$\hat{X} = G^{-1} H^T R^{-1} Z \quad \textit{Direct solution}$$

$$G = H^T R^{-1} H \; ; R = E\{\upsilon \cdot \upsilon^T\} = \mathrm{cov}(\upsilon)$$

$$\sigma_i^2 : R(i,i) \; \textit{error variance}$$

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Phasor-only WLS state estimation

Consider a fully measured system:

$$Z^m = \begin{bmatrix} V^m \\ I^m \end{bmatrix} \quad \begin{matrix} \Rightarrow \textit{Bus voltages} \\ \Rightarrow \textit{Branch currents} \end{matrix}$$

$$= \begin{bmatrix} U \\ Y_b \cdot A \end{bmatrix} \cdot [V] + \upsilon$$

U : identity matrix

Y_b : branch admittance matrix

A : branch - bus incidence matrix

Note: Shunt branches are neglected initially, they will be introduced later.

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Phasor-only WLS state estimation

$$Let \begin{bmatrix} V^m \\ I^m \end{bmatrix} = \begin{bmatrix} e^m + jf^m \\ c^m + jd^m \end{bmatrix}$$

$$= \begin{bmatrix} U \\ (g + jb) \cdot A \end{bmatrix} \cdot \left[e + jf \right] + \upsilon$$

$$= \left[H_F \right] \cdot \left[e + jf \right] + \upsilon$$

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Phasor-only WLS state estimation: Complex to real transformation

$$\begin{bmatrix} e^m \\ f^m \\ c^m \\ d^m \end{bmatrix} = \begin{bmatrix} U & \\ & U \\ gA & -bA \\ bA & gA \end{bmatrix} \cdot \begin{bmatrix} e \\ f \end{bmatrix} + \upsilon$$

$$[Z] = [H] \cdot \begin{bmatrix} e \\ f \end{bmatrix} + \upsilon$$

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Phasor-only WLS state estimation: Exact cancellations in off-diagonals of [G]

R is assumed to be identity matrix without loss of generality

$$G = H^T \cdot H = \begin{bmatrix} U & \\ & U \\ gA & -bA \\ bA & gA \end{bmatrix}^T \cdot \begin{bmatrix} U & \\ & U \\ gA & -bA \\ bA & gA \end{bmatrix}$$

$$= \begin{bmatrix} U + A^T\left(g^T g + b^T b\right)A & 0 \\ 0 & U + A^T\left(b^T b + g^T g\right)A \end{bmatrix}$$

[G] matrix:

- Is block – diagonal
- Has identical diagonal blocks
- Is constant, independent of the state

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Phasor-only WLS state estimation: Correction for shunt terms

$$[Z] = (H + H_{sh}) \cdot X + \upsilon = H \cdot X + u$$

$$u = H_{sh} \cdot X + \upsilon$$

$$E\{u\} = H_{sh} \cdot E\{X\}$$

$$E\{X\} = \hat{X} = G^{-1} H^T R^{-1} Z$$

$$X^{corr} = G^{-1} H^T R^{-1} (Z - H_{sh} \cdot \hat{X})$$

$$= \hat{X} - \underbrace{G^{-1} H^T R^{-1} H_{sh}}_{\text{Very sparse}} \cdot \hat{X}$$

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Fast Decoupled WLS Implementation Results

Test Systems Used

System Label	Number of Buses	Number of Branches	Number of Phasor Measurements
A	159	198	222
B	265	340	361
C	3625	4836	4982

Cases simulated:

Case-1: No bad measurement.

Case-2: Single bad measurement.

Case-3: Five bad measurements.

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Fast Decoupled WLS Implementation Results

Average MSE Values (for 100 Simulations)

System	Case	MSE	
		WLS	Decoupled WLS
A	1	0.59	0.59
	2	0.59	0.59
	3	0.62	0.62
B	1	0.61	0.61
	2	0.61	0.60
	3	0.62	0.63
C	1	0.59	0.59
	2	0.58	0.59
	3	0.58	0.59

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Fast Decoupled WLS Implementation Results

MEAN CPU TIMES OF 100 SIMULATIONS

System	Case	CPU Times (ms)	
		WLS	Decoupled WLS
A	1	5	2.4
	2	5.7	2.7
	3	9.3	3.9
B	1	7.5	3.5
	2	8.7	3.9
	3	14.8	5.8
C	1	137.4	75.9
	2	169.5	95.7
	3	284.7	165.6

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L_1 (LAV) Estimator

$$Minimize \ \sum_{i=1}^{m} c^T \cdot |r|$$

$$Subject\ to \ \ Z = H \cdot \hat{X} + r$$

$$c^T = [1,1,\ldots,1]$$

Robust but with known deficiency:

Vulnerable to **<u>leverage</u>** measurements

Leverage points in measurement model

Mili L., Cheniae M.G., and Rousseeuw P.J., "Robust State Estimation of Electric Power Systems" IEEE Transactions on Circuits and Systems, Vol. 41, No. 5, May 1994, pp.349-358.

- Flow measurements on the lines with impedances, which are very different from the rest of the lines.
- Using a very large weight for a specific measurement.

$$H\ x + e = z$$

$$R^{-1}H = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \\ x & x & x \\ Y & Y & Y \\ x & x & x \end{bmatrix}$$

LEVERAGE MEASUREMENT

Scaling both the measured value and the measurement jacobian row will eliminate leveraging effect of the measurement.

Leave zero injections since they are error free by design. Incorporate equality constraints in the formulation.

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Properties of L_1 estimator

- Efficient Linear Programming (LP) code exists to solve it for large scale systems.
- Use of simple scaling eliminates leverage points. This is possible due to the type of phasor measurements (either voltages or branch currents).
- L_1 estimator automatically rejects bad data given sufficient local redundancy, hence bad data processing is built-in.

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Bad data processing

WLS: Post estimation bad data processing / re-estimation

$\Omega = R - HG^{-1}H^T$ Normalized Residuals Test (NRT):

$G = H^T R^{-1} H$

$r_i^N = |r_i| / \sqrt{\Omega_{ii}}$ → $z_i^{new} = z_i^{bad} - R_{ii} / \Omega_{ii} \, r_i^{bad}$ → Update the estimates

NRT is repeated as many times as the number of bad data.

L_1: Linear Programming Solution

LP problem is solved once. Choice of initial basis impacts solution time/iterations.

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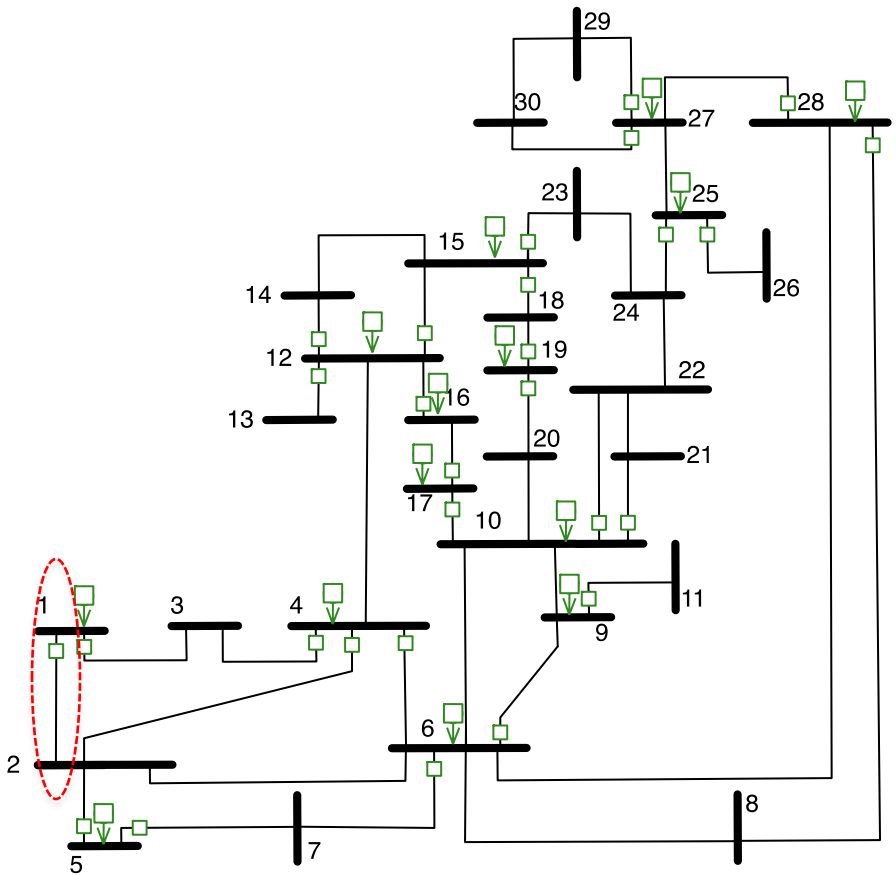
Small system example

IEEE 30-bus system



Line 1-2 parameter is changed to transform I_{1-2} into a leverage measurement

Case 1: Base case true solution

Case 2: Bad leverage measurement without scaling

Case 3: Same as Case 2, but using scaling

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Small system example

Base case
True Solution

Leverage Measurements
Bad data
No scaling

Leverage Measurements
Bad data
With scaling

	Case-1		Case-2		Case-3	
	V (pu)	**θ (deg)**	**V (pu)**	**θ (deg)**	**V (pu)**	**θ (deg)**
1	1	5.2	1.0118	-3.5462	1	5.2
2	1	-3.74	0.9999	-3.74	1	-3.74
3	0.9952	0.13	1.004	-3.8964	0.9952	0.13
4	0.9992	-4.81	0.9992	-4.81	0.9992	-4.81
5	1	-9.02	0.9999	-9.02	1	-9.02
6	0.9851	-7.41	0.9851	-7.41	0.9851	-7.41
7	0.9869	-8.88	0.9869	-8.88	0.9869	-8.88

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Simulation Results

3625 bus, 4836 branch utility system

	CPU Time (seconds)		
	No Bad Data	Single Bad Data	Five Bad Data
LAV with built-in BD removal	3.33	3.36	3.57
WLS using post-SE BD detection test	2.32	9.38	50.2

- Bad data handling of LAV solver remains fairly insensitive to the number of bad data.
- Bad data handling of WLS solver will be proportionally slower with increasing number of bad data in the measurement set.

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Remarks on CPU Results

- WLS SE solution is faster when there are no bad data.
- Under bad data LAV SE performs comparable or better than WLS SE. Advantages become more pronounced with increasing bad data.
- WLS needs some form of post-SE bad data processor (e.g. largest normalized residual test) to detect and remove bad data.

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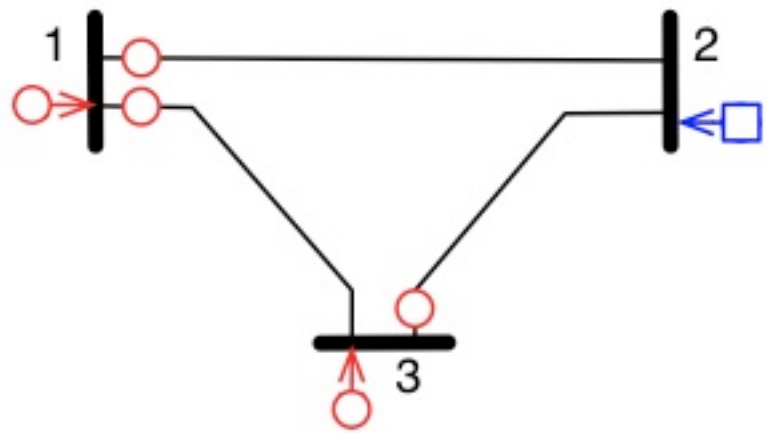
SCADA Based Implementation



Measurement set:

- Power injection measurements
- Power flow measurements
- Voltage magnitude measurements

Weighted Least Squares (WLS) Estimator

- Well-developed and widely-known
- Requires bad-data analysis (non-robust)
  - Normalized residuals test
  - Re-weighting

Least Absolute Value (LAV) Estimator

- Linear programming based, computationally inefficient
- Does not require bad-data analysis
- Deficiency in the presence of leverage measurements

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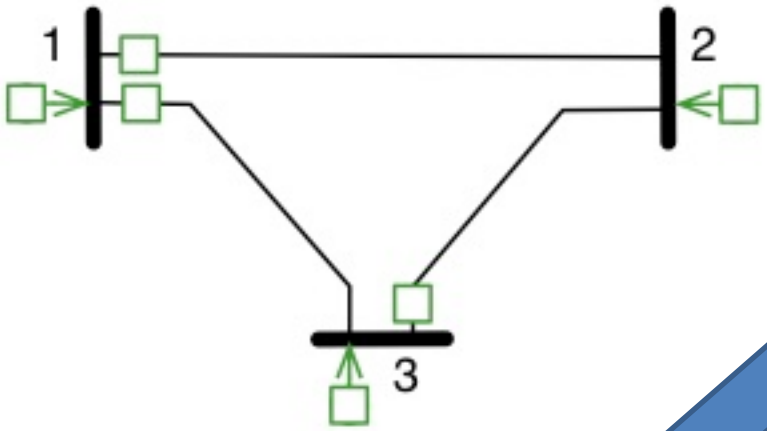
PMU Based Implementation



Phasor Measurement Units (PMUs):

Linearly related states and measurements.

WLS :

- Linear solution
- Requires bad-data analysis
  - Normalized residuals test
  - Re-weighting (not applicable)
- No deficiency in the presence of leverage measurements, with *scaling*.
- Exact cancellations in [G]

LAV (robust):

- Linear programming (single step), *computationally efficient*
- Does not require bad-data analysis
- No deficiency in the presence of leverage measurements, with *scaling*.

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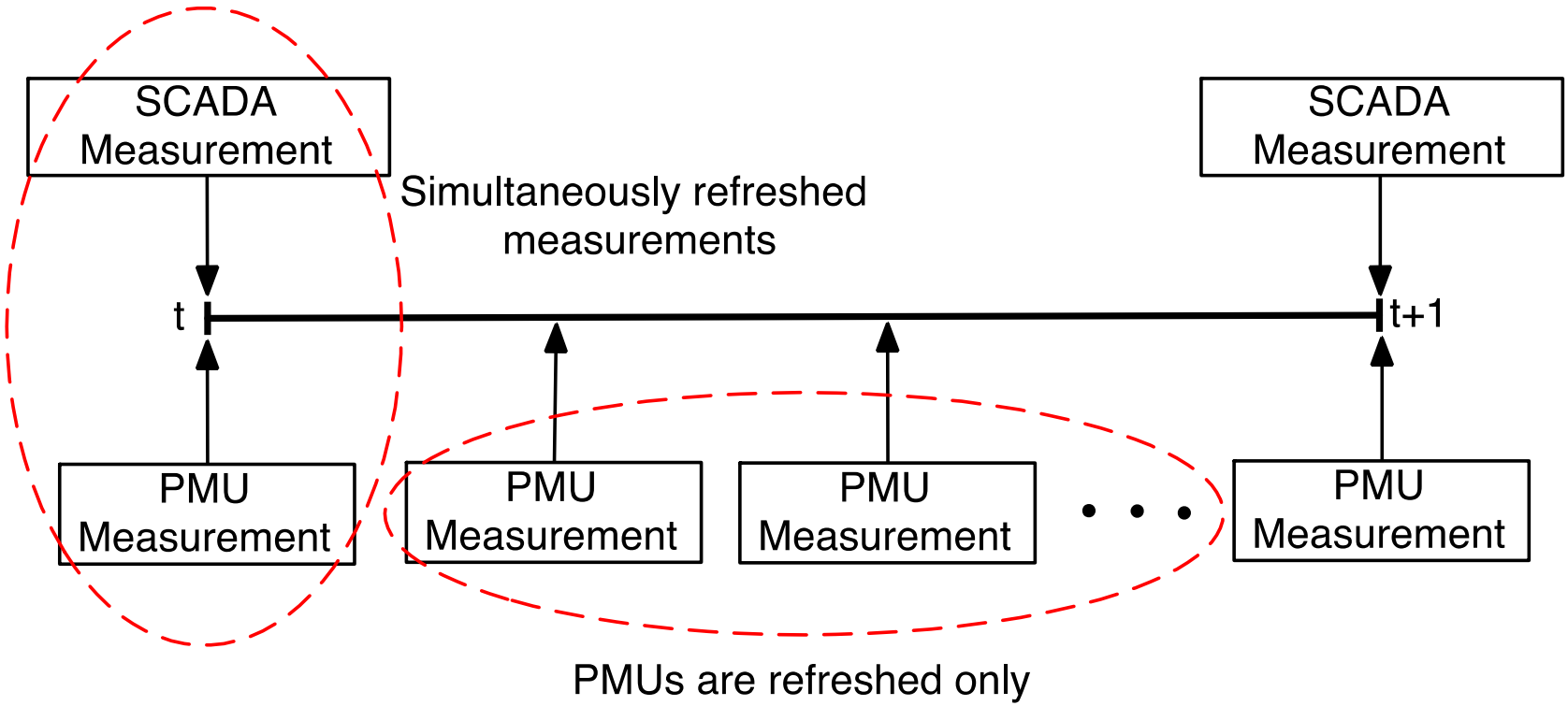
Hybrid SE Using Both SCADA and PMU Measurements

Main challenge:
Different refresh rates of SCADA and PMU measurements.



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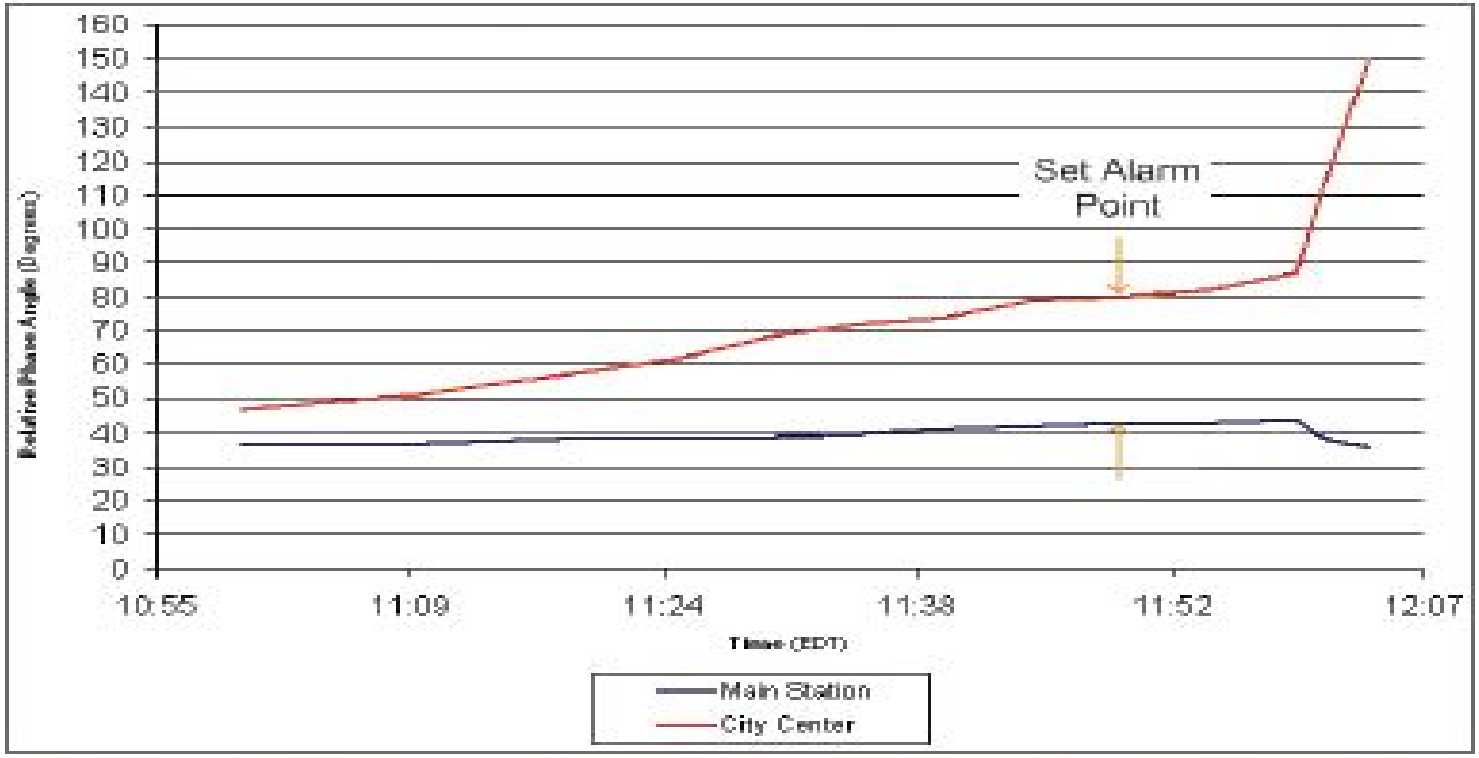
Re: SEL Synchrophasors Brochure



Phase angle measurement between critical locations on a power system provides operators with an early warning of potential system collapse.

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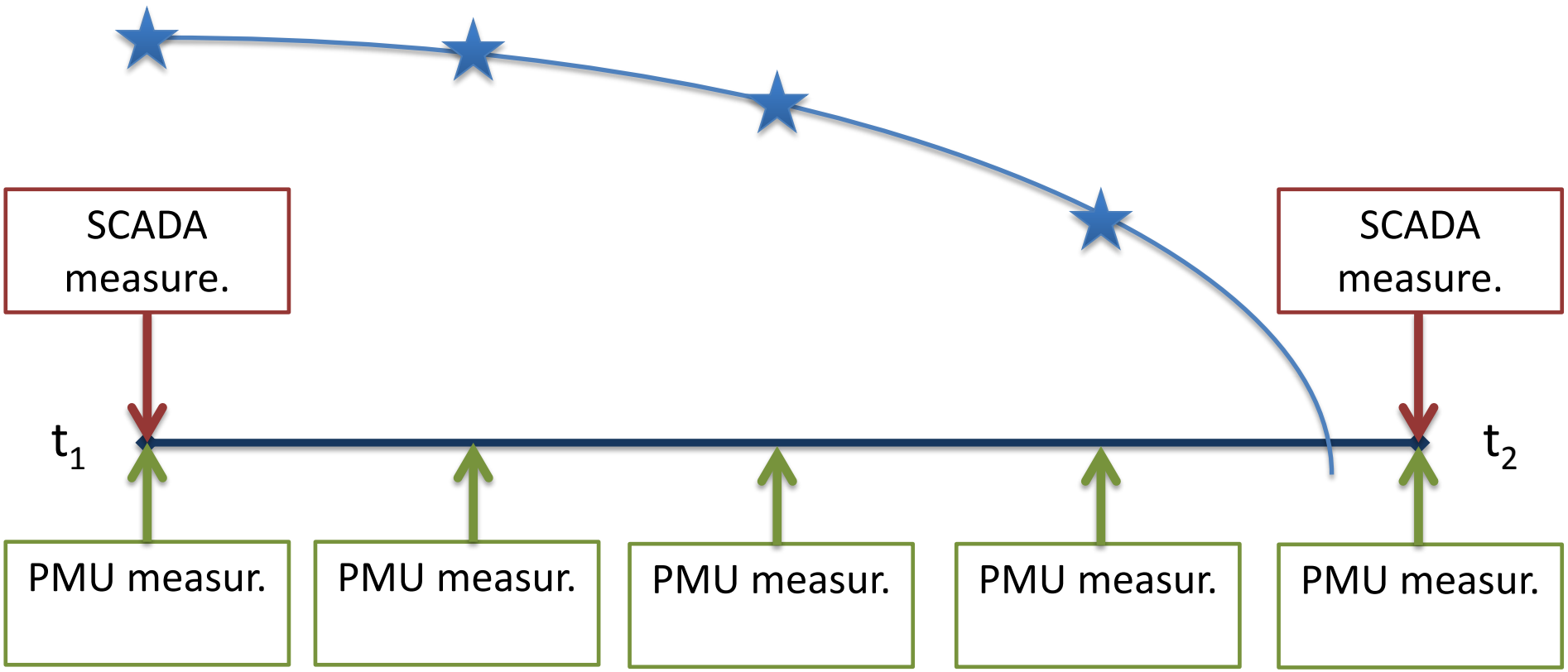
SCADA and PMU Measurements

Using conventional SCADA-based SE → System collapses without warning
Using mixed measurement based SE → Tracks state and takes control action



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Hybrid State Estimator

Must be able to process both PMU and SCADA measurements simultaneously.

At instances when both types of measurements are received it uses a WLS estimator.

Otherwise, it switches to a robust Least Absolute Value (LAV) based state estimator.

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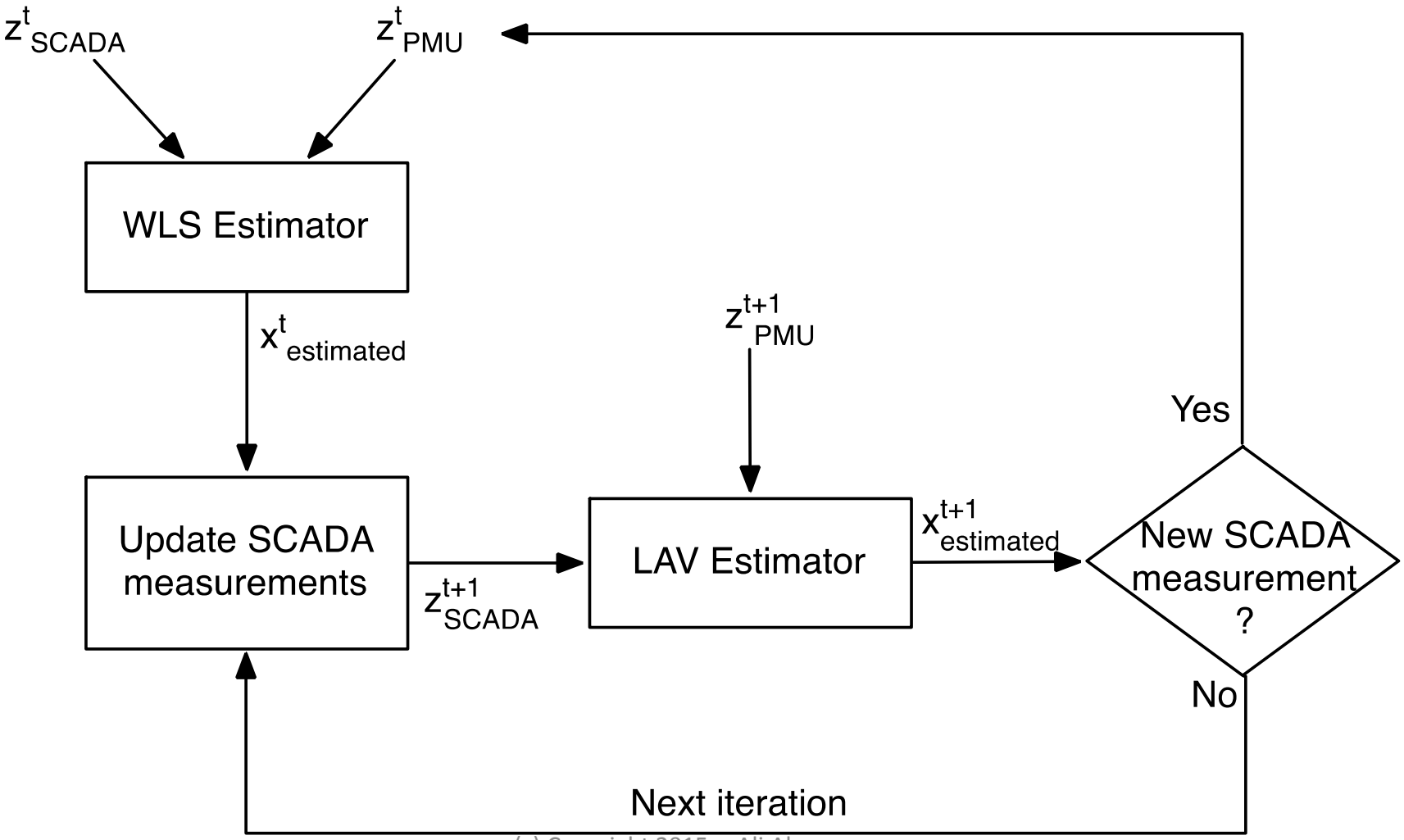
Flowchart of Hybrid SE



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Implementation

Linearize measurement equations

$$\Delta z = z^k - h(x^k)$$

$$\Delta x = x^{k+1} - x^k$$

$$\Delta z = H\Delta x + e$$

Update SCADA measurements [*]

$$z_{SCADA}^{k+1} = h_{SCADA}(x^k)$$

[*] M. Glavic and Th. Van Cutsem, "Tracking network state from combined SCADA and synchronized phasor measurements ," IREP 2013.

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Test Case: Voltage Collapse

- IEEE 57 bus system
  - 9 branch PMUs
  - 32 Power injection measurements
  - 32 Power flow measurements
- Voltage collapse at bus 22
  - No PMU at the bus.

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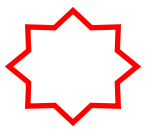
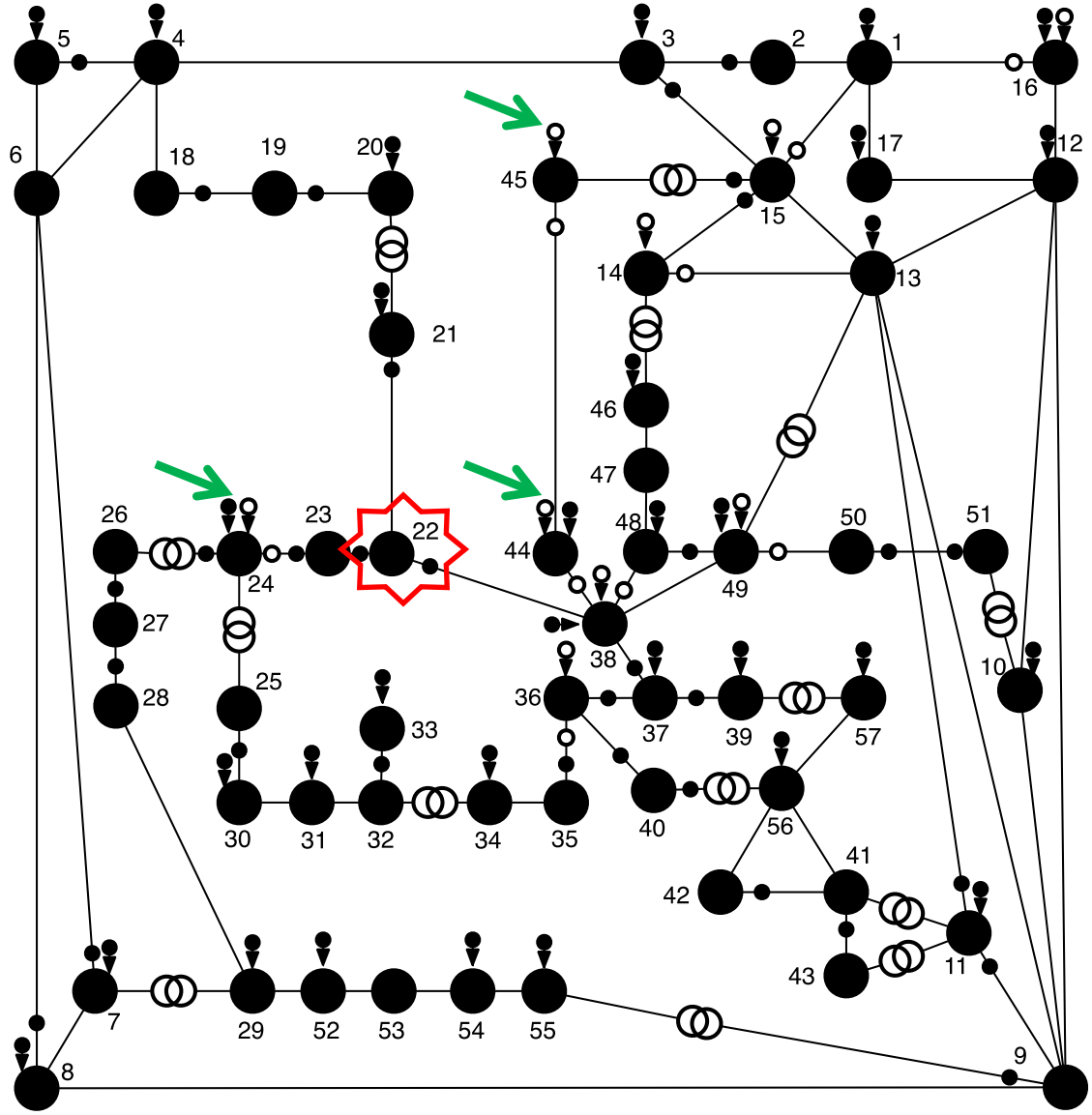
57-Bus Example



Voltage Collapse



Power injection measurement
Power flow measurement
Voltage phasor measurement
Current phasor measurement

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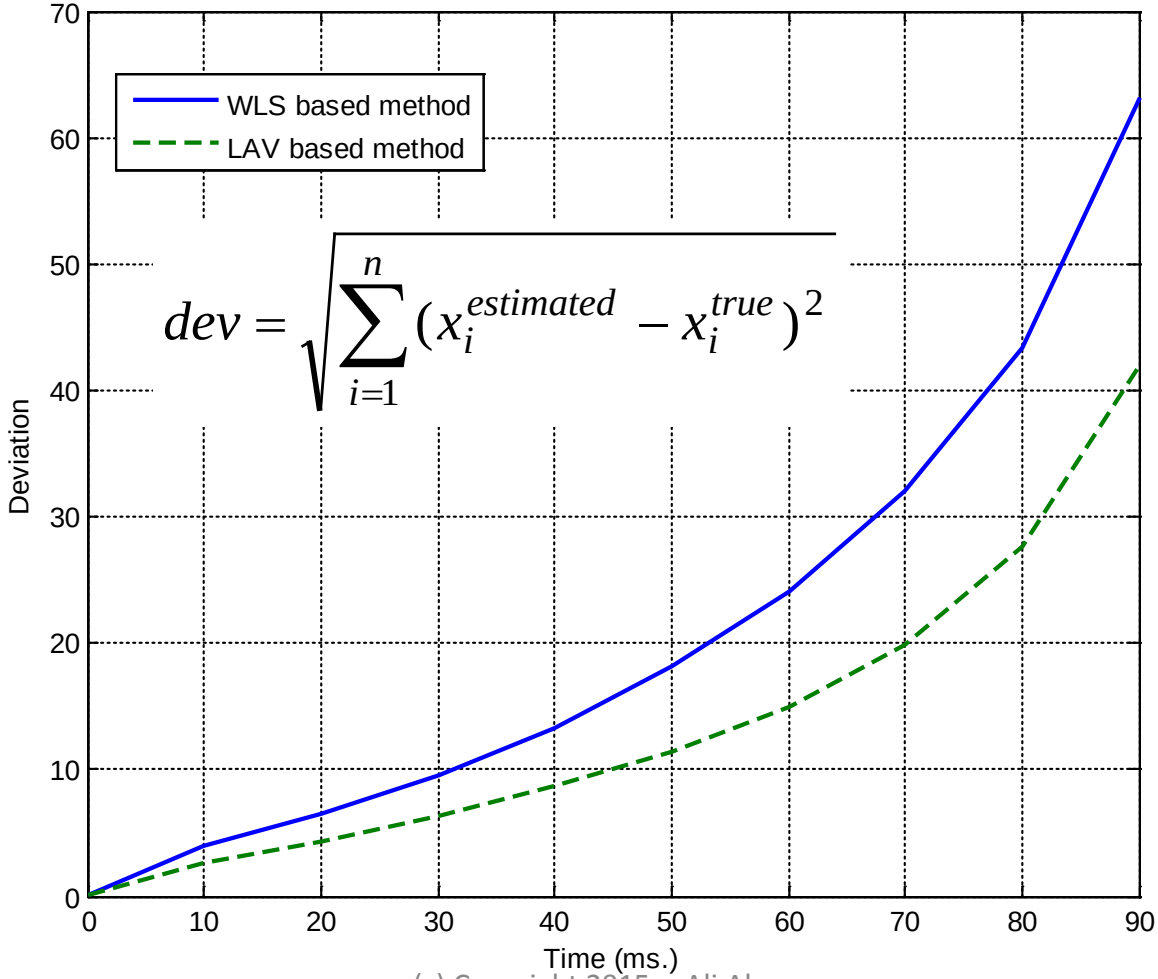
Comparison of the two estimators

[WLS based updates:]
M. Glavic and Th. Van Cutsem, “Tracking network state from combined SCADA and synchronized phasor measurements ,” IREP 2013.



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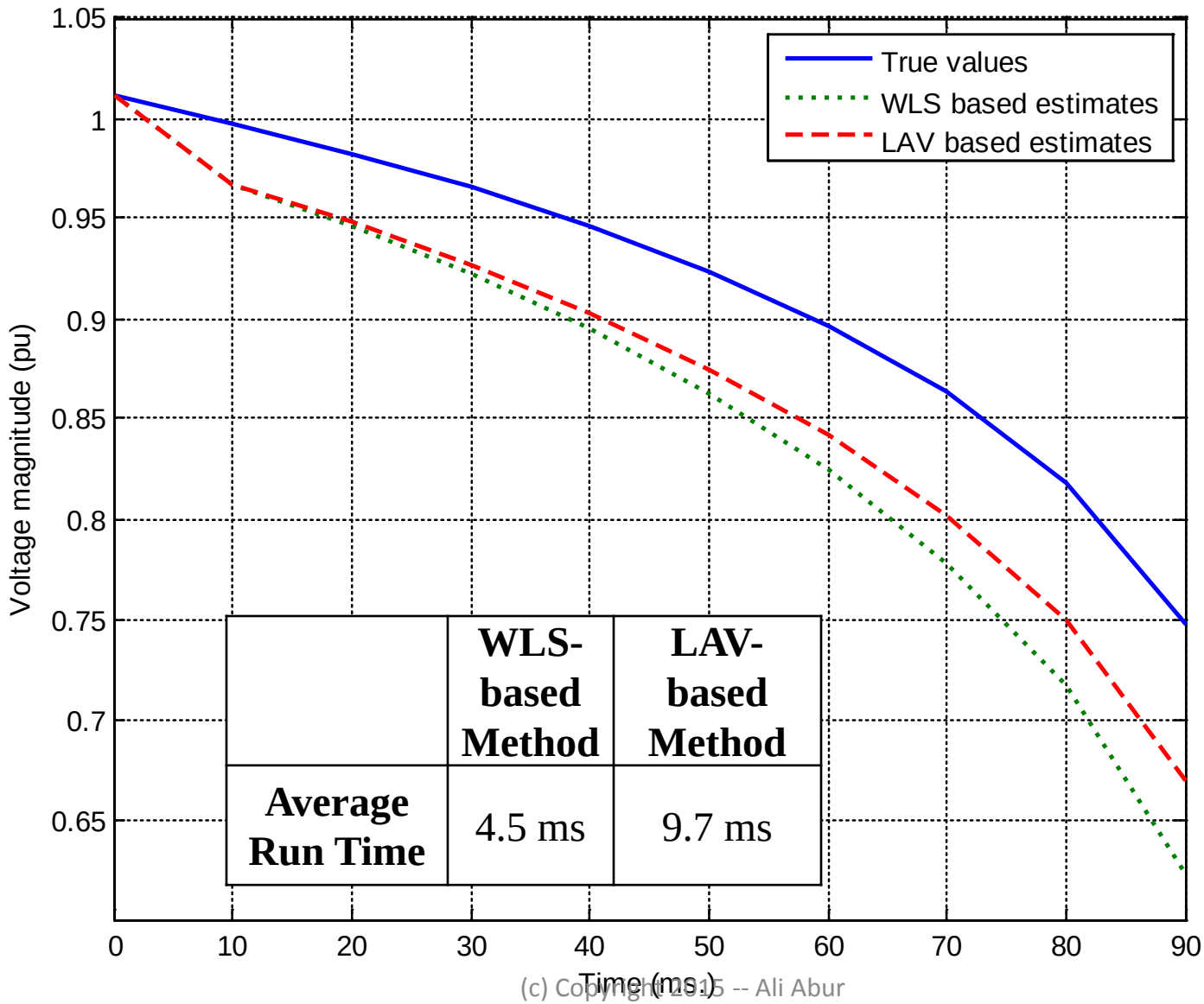
Voltage at Bus 22 Tracked by the Two Estimators



	WLS-based Method	LAV-based Method
Average Run Time	4.5 ms	9.7 ms

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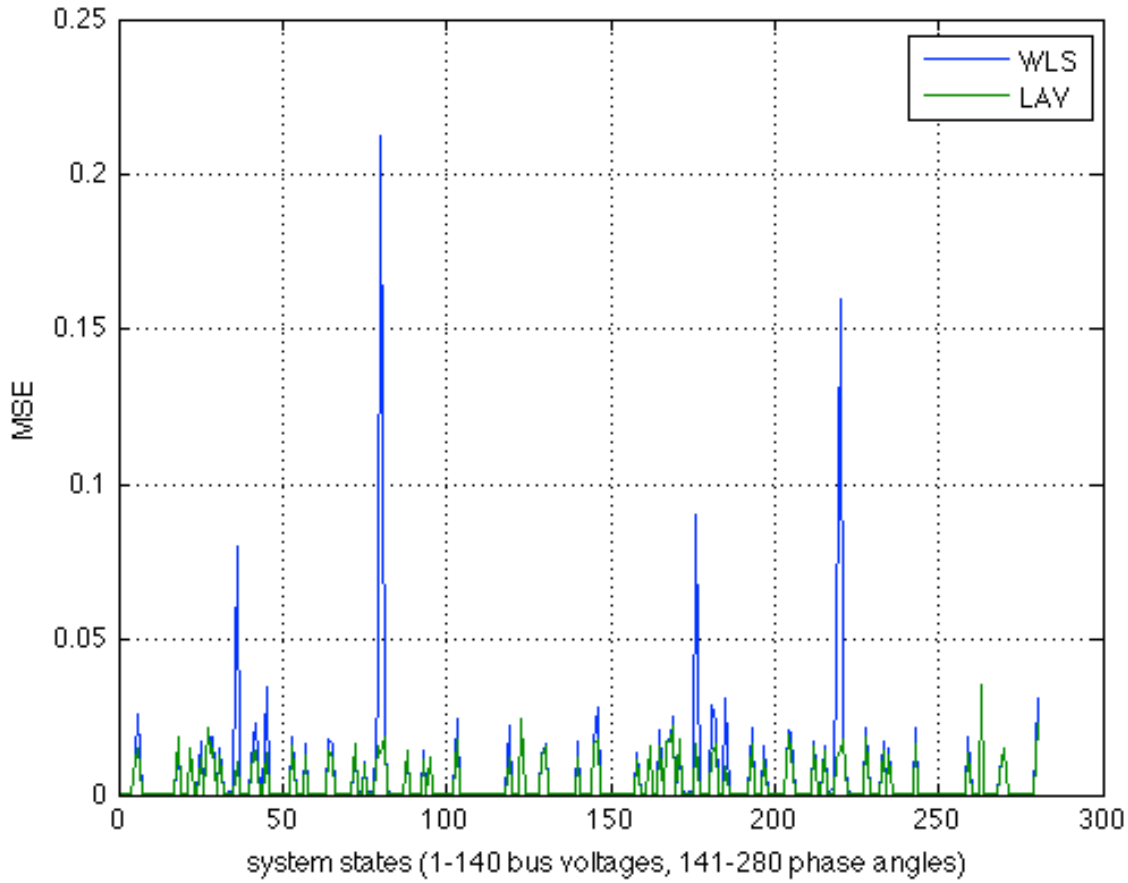
Simulation Results (No Bad Data)

140-bus, 233-branch NPCC system.
95 power injections and 205 power flows
SCADA/sec and PMU*30/sec; T=10 sec.

$$MSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i^{estimated} - x_i^{true})^2}$$

90 PMUs



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Simulation Results (with Bad Data)

For the entire simulation:

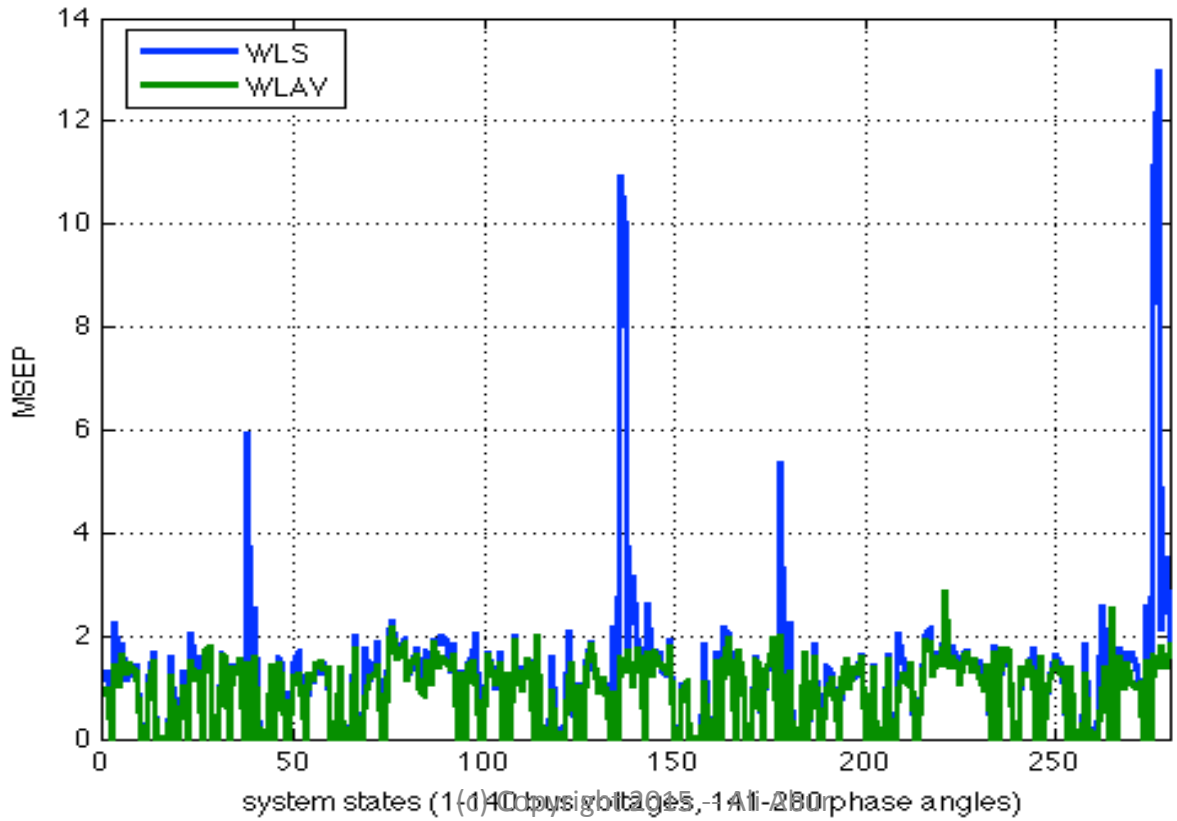
$P_{inj\ \_126} = 0$

$P_{flow\ \_121\text{-}119} = 0$

Assume 30 PMUs are installed in the system.

Percentage Mean Squares Error

$$MSEP = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(100 \times \left(x_i^{estimated} - x_i^{true}\right) \Big/ x_i^{true}\right)^2}$$



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3-phase Operation: Motivation and Observations

- State estimators implicitly assume balanced operating conditions and use positive sequence network model and measurements. This assumption is sometimes questionable.
- Phasor measurements are linearly related to the states, thus can be decomposed into modal components.
- It will then be possible to utilize existing, well developed and tested software to run state estimation in each mode independently and in parallel.

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Proposed approach: Modal decomposition revisited

- Decompose phasor measurements into their symmetrical components
- Estimate individual symmetrical components of states separately (in parallel if such hardware is available)
- Transform the estimates into phase domain to obtain estimated phase voltages and flows.

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Modal decomposition of measurement equations

$$Z = HV + e$$

$$Z^T = [Z_V^T Z_I^T]$$

Phasor domain measurement representation
H: 3mx3n
V: 3nx1
Z: 30x1

$$V_S = TV_P \quad \& \quad I_S = TI_P$$

$$T_Z = \begin{bmatrix} T & 0 & \cdots & 0 \\ 0 & T & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & T \end{bmatrix}$$

T: 3mx3m

Modal domain vectors
T: 3x3
V_S and I_S: 3x1

$$T_Z Z = T_Z HV + T_Z e$$

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Modal decomposition of measurement equations

$$V = T_V V_M \quad \Longrightarrow \quad T_V = \begin{bmatrix} T^{-1} & 0 & \cdots & 0 \\ 0 & T^{-1} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & T^{-1} \end{bmatrix}$$

$$Z_M = H_M V_M + e_M$$

$$Z_M = T_Z Z$$
$$H_M = T_Z H T_V$$
$$e_M = T_Z e$$

Zero sequence

$$Z_0 = H_0 V_0 + e_0$$
$$V_0 = G_0^{-1} H_0^T R_0^{-1} Z_0$$
$$G_0 = H_0^T R_0^{-1} H_0$$

Positive/ Negative sequence

$$Z_r = H_r V_r + e_r$$
$$V_r = G_r^{-1} H_r^T R_r^{-1} Z_r$$
$$G_r = H_r^T R_r^{-1} H_r$$

!

Z_0, Z_r : mx1
V_0, V_r: nx1
H_0, H_r: mxn

- Fully decoupled three relations
- Smaller size (Jacobian) matrices

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Large scale 3-phase system example: 3625-buses and 4836-branches

CPU (seconds)	No Bad Data		One Bad Datum		Five Bad Data	
	LAV	WLS	LAV	WLS	LAV	WLS
Zero Seq.	3.52	2.32	3.65	9.28	3.78	25.51
Positive Seq.	3.61	2.62	3.63	9.41	3.66	26.01
Negative Seq.	3.21	2.22	3.45	9.01	3.62	24.82

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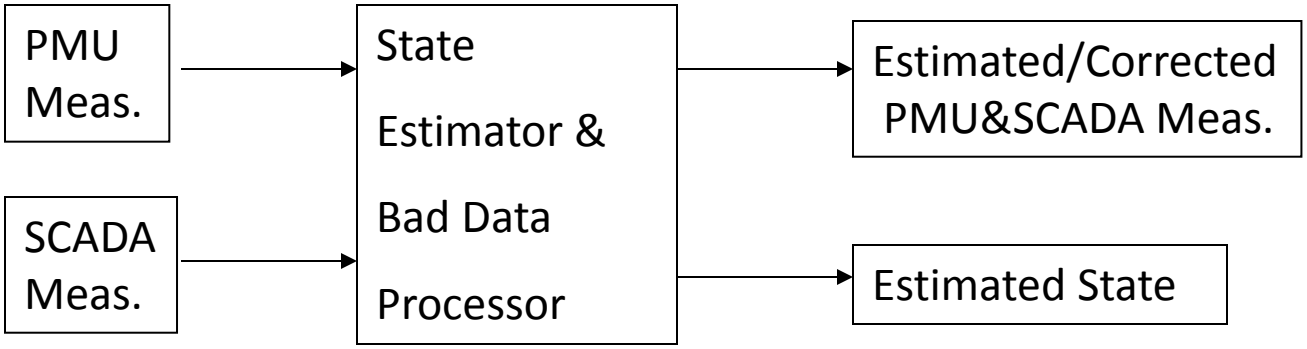
No Reference Bus is Needed

- Eliminate the reference phase angle from the SE formulation.
- Bad data in SCADA as well as phasor measurements can be detected and identified with sufficiently redundant measurement sets.



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Final Remarks

- SE performance have significant impact on all other application functions related to the transmission grid
- PMUs can improve SE performance in the following areas:
  - Computational speed and numerical robustness
  - 3-Phase state estimation

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Thank You
Any Questions?

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